NMR-like control of a quantum bit superconducting circuit

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Different nanofabricated superconducting circuits based on Josephson junctions have already achieved a degree of quantum coherence sufficient to demonstrate coherent superpositions of their quantum states. These circuits are considered for implementing quantum bits, which are the building blocks for the recently proposed quantum computer designs. We report experiments in which the state of a quantum bit circuit, the quantronium, is efficiently manipulated using methods inspired from Nuclear Magnetic Resonance (NMR).

Despite progress in the development of quantum bit (qubit) electronic circuits, the complexity and robustness of the operations that have been performed on them are still too primitive for implementing a small quantum processor circuit that would demonstrate the power of quantum computing (see [1] for a review). Presently, the most advanced qubit circuits are superconducting ones based on Josephson junctions. The preparation of coherent superpositions of the two states of a qubit has already been achieved for several circuits [2, 3, 4, 5, 6, 7, 8], and a two qubit gate was recently demonstrated [9]. The coherence time of the superpositions prepared is however short, which explains why qubit transformations are far less developed for qubit circuits than for microscopic quantum systems such as atoms or spins. In this Letter, we report on experiments that successfully manipulate a Josephson qubit based on the quantronium circuit [10], using methods developed in NMR. We demonstrate that any transformation of the qubit can be implemented, that they can be made robust against detuning effects and that decoherence due to noise in the control parameters can be fought.

The quantronium circuit, described in Fig. 1, is derived from the Cooper pair box [11, 12]. It consists of a superconducting loop interrupted by two adjacent small Josephson tunnel junctions with capacitance $C_j/2$ and Josephson energy $E_J/2$ each, and by a larger Josephson junction ($E_{J0} \approx 15E_J$) for readout. The two small junctions define a low capacitance C_Σ superconducting electrode called the "island", with charging energy $E_C = (2e)^2/2C_\Sigma$. This island is biased by a voltage source U through a gate capacitance C_g. Fabrication is performed using standard e-beam lithography, and the characteristic energies measured in the sample reported in this work are $E_J = 0.87 \ k_B$ K and $E_C = 0.66 \ k_B$ K. Experiments are performed at 20 mK using filtered connecting lines. The eigenstates of this system are determined by the dimensionless gate charge $N_g = C_g U/2e$, and by the superconducting phase $\delta = \gamma + \phi$ across the two small junctions, where γ is the phase across the large junction and $\phi = \Phi/\varphi_0$, with Φ the external flux imposed through the loop and $\varphi_0 = \hbar/2e$. For a large range of parameters, the spectrum is anharmonic, and the two lowest energy states $|0\rangle$ and $|1\rangle$ form a two-level system suitable for a qubit. At the optimal working point ($\delta = 0, N_g = 1/2$), the transition frequency ν_{01} is stationary with respect to changes in the control parameters, which makes the quantronium immune to phase and charge noise [3, 10]. For the sample investigated, $\nu_{01} \simeq 16.40$ GHz at the optimal working point. For readout, $|0\rangle$ and $|1\rangle$ are discriminated through the difference in their supercurrents in the loop [3]. A trapezoidal readout pulse $I_b(t)$ with a peak value slightly below the critical current $I_0 = E_{J0}/\varphi_0 \simeq 550$ nA is applied to the circuit so that, by adjusting the amplitude and duration of the pulse, the switching of the large junction to a finite voltage state is induced with a large probability p_1 for state $|1\rangle$ and with a small probability p_0 for state $|0\rangle$. The switching is detected by measuring the voltage across the readout junction with a room temperature amplifier, and the switching probability p is determined by repeating the experiment a few 10^4 times at a rate $10 - 60$ kHz. The fidelity of the measurement is the largest value of $\eta = p_1 - p_0$.

The manipulation of the qubit state is achieved by applying time dependent control parameters $N_g(t)$ and $I_b(t)$. When a nearly resonant microwave voltage is applied to the gate, the Hamiltonian is conveniently described using the Bloch sphere in a frame rotating at the microwave frequency. When the gate charge N_g varies as $N_g(t) = N_{g0} + \Delta N_g \cos(2\pi\nu_{\mu w}t + \chi)$, where χ is the phase of the pulse with respect to the microwave reference, the hamiltonian $h = -\vec{H} \cdot \overrightarrow{\sigma}/2$ is that of a spin 1/2 in an effective magnetic field $\vec{H} = h\Delta\nu \, \vec{z} + h\nu_{R0} [\vec{x}\cos\chi + \vec{y}\sin\chi]$, where $\Delta\nu = \nu_{\mu w} - \nu_{01}$ is the detuning, and $\nu_{R0} = 2E_C\Delta N_g \langle 1| \widehat{N} |0\rangle /h$ the Rabi frequency. At $\Delta\nu = 0$, Rabi precession takes place around an axis lying in the equatorial plane, at an angle χ with respect to the X axis. Rabi precession between the qubit states induces oscillations of the switching probability p with the pulse duration, as shown in Fig. 1, at a Rabi frequency that scales with the amplitude ΔN_g [3]. The range of Rabi frequencies ν_{R0} that could be explored extends above 250 MHz, and the shortest π pulse duration for preparing $|1\rangle$ starting from $|0\rangle$ was less than 2 ns. The fidelity was $\eta \simeq 0.3 - 0.4$ for readout pulses with 100 ns duration. It was obtained with the readout performed at a value of the phase $\langle\delta\rangle$ where the difference between the loop currents for the two qubit states is maximised. The fidelity is smaller than expected, possibly due to spurious relaxation of the qubit during the readout process, and

might be improved using rf methods that avoid switching to the voltage state [13].

In order to demonstrate that arbitrary operations on the qubit are possible, it is necessary to combine rotations around different axes. In Fig 2, measurements of the switching probability p following two-pulse sequences combining $\pi/2$ rotations around the axes X, Y, $-X$, or $-Y$, are shown. The theory predicts that p oscillates at frequency $\Delta\nu$ with the delay Δt between pulses. This experiment is analogous to the Ramsey experiment in atomic physics, and to the free induction decay in NMR. When the two pulses have different phases χ_1 and χ_2, the Ramsey pattern is phase shifted by $(\chi_2 - \chi_1)$. Despite the presence of spurious frequency jumps due to individual charge fluctuators near the island, the overall agreement for the phase shift of the Ramsey pattern demonstrates that rotations around axes X and Y combine as predicted. Arbitrary rotations of the qubit can thus be performed by combining three rotations around these axes [1]. However, rotations around the Z axis can be more readily performed by changing the qubit frequency for a short time. Note that frequency agility is also interesting when interacting qubits need to be tuned at the same transition frequency. In order to change the frequency, a triangular bias current pulse with maximum amplitude ΔI is applied during a Ramsey experiment. During the pulse, a phase difference $\zeta = 2\pi \int \delta\nu_{01}(t)dt$ builds up between the qubit states, which is equivalent to a rotation around the Z axis with an angle ζ. The Ramsey pattern is phase-shifted by ζ as shown by the oscillations of p with ΔI (right panel in Fig 3). In the fit, the rotation angle is calculated using the measured transition frequency as a function of δ. Rotations around the Z axis have also been performed using adiabatic pulses in N_g.

Although these rotation experiments demonstrate that arbitrary unitary transformations of the qubit state can be performed, their accuracy and robustness are still issues. Similar questions have arisen in NMR, and methods have been developed there to make spin manipulations less sensitive to rf pulse imperfections. The transformation fidelity has furthermore been addressed with the advent of NMR-based quantum computing [1]. It has been shown in particular that composite pulses can provide an appreciable improvement [15]. In the case of the so-called $CORPSE$ sequence (Compensation for Off-Resonance with a Pulse Sequence), a single pulse is replaced by three, with the advantage of a strongly reduced sensitivity to detuning. We have tested this sequence in the case of a π rotation around the X axis, which performs a NOT operation on the qubit. The corresponding $CORPSE$ $\pi(X)$ pulse sequence is $\{7\pi/3(X), 5\pi/3(-X), \pi/3(X)\}$ [15]. As shown in Fig. 4, this sequence is significantly more robust against detuning than a single pulse $\pi(X)$ since the switching probability stays close to its maximum value over a larger frequency span of about 100 MHz, comparable to the Rabi frequency $\nu_{R0} = 92$ MHz. By performing the $CORPSE$ sequence after an arbitrary rotation $\theta(-X)$, we have also checked that the $CORPSE$ sequence works for a general initial state.

The decay of Ramsey oscillations at long times provides a direct measurement of the coherence time T_2. This decay was close to exponential with $T_2 = 300 \pm 50$ ns for the present sample at the optimal working point (see top panel in Fig. 5), and became progressively faster when departing from that point [3]. These observations validate the concept of optimal working point, which has also been exploited recently in flux qubits [14]. The coherence time is limited by relaxation and dephasing. Relaxation can be fought by better balancing the two small josephson junctions [3, 4]. In the sample reported in this work, the relaxation time T_1 was 500 ± 50 ns at the optimal working point, where dephasing due to low frequency charge and phase noises, which dominates decoherence, is minimum. However, coherence times T_2 of the order of a few hundreds of nanoseconds are still too short to implement quantum algorithms, and increasing the coherence time is a major concern, for which concepts of NMR are again useful. First, the well known spin-echo technique [16] allows to suppress part of the dephasing. By inserting a π pulse in the middle of a Ramsey sequence, the random phases accumulated during the two free evolution periods before and after the π pulse cancel provided that the perturbation is almost static on the time-scale of the sequence. The echo method provides a simplified form of error correction that suppresses the effect of low frequency fluctuations. As shown in Fig. 5 (middle panel), the decay time of echoes is indeed longer than that of the Ramsey pattern. At the optimal point, the echo decay time is roughly doubled to 550 ± 50 ns. Away from the optimal point in the charge direction N_g, this decay time is maintained at about 500 ns till T_2 falls below 10 ns. Moving away in the phase direction δ, the echo compensation is less efficient, and the echo decay time is roughly $2T_2$ till T_2 falls below 20 ns. The efficiency of echo compensation provides a probe of dephasing mechanisms, and thus of noise sources.

Another way to increase the effective coherence time is to continuously drive the qubit with a microwave signal, a method called spin-locking in NMR [16]. A spin-locking sequence consists of a Ramsey sequence, but with a driving locking field along the direction Y or $-Y$ continuously applied between the two $\pi/2(X)$ pulses. Since the state $|-Y\rangle = (|0\rangle - i|1\rangle)/\sqrt{2}$ prepared by the first pulse is then an eigenstate of the Hamiltonian in the rotating frame in presence of the locking field, it does not evolve in time. Relaxation between the states $|-Y\rangle$ and $|Y\rangle$ occurs under the effect of fluctuations occuring at the Rabi locking frequency, at a rate called the relaxation rate in the rotating frame in NMR. As shown in Fig. 5 (bottom panel), the decay time of p after the two spin-locking sequences $\{\pi/2(X), Lock(Y), \pm\pi/2(X)\}$, is equal to 650 ± 50 ns, which is significantly longer than T_2. This decay time furthermore does not depend on the orientation of the locking field along Y or $-Y$ because the energy difference between the states $|Y\rangle$ and $|-Y\rangle$ in the rotating frame is $h\nu_{R0} \ll kT$. Spin-locking provides a weak form of error correction because low frequency fluctuations of the hamiltonian are followed adiabatically by the eigenstates. This

shows that applying continuously a driving field can suppress part of decoherence experienced by a qubit during its free evolution.

In conclusion, we have demonstrated that the state of a quantronium qubit can be efficiently manipulated using methods inspired from NMR. Rotations around X and Y axes with microwave pulses have been combined, rotations around the Z axis have been performed with adiabatic pulses, and robust rotations have been performed using composite pulses. Finally, the spin-echo and spin-locking methods have yielded a significant increase of the effective coherence time of the qubit. The quantitative investigation of qubit decoherence using NMR techniques will be the subject of further work.

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FIG. 1: Top: circuit diagram of the "quantronium". The Hamiltonian of this circuit is controlled by the gate-charge N_g applied to the island between the two small junctions, and by the phase δ across their series combination. This phase is determined by the flux ϕ through the loop, and by the bias-current I_b. The two lowest energy states form a two level system suitable for a quantum bit. The readout of the qubit state is performed by inducing the switching of the larger readout junction to a finite voltage V with a bias-current pulse $I_b(t)$ approaching the critical current of this junction. Bottom: The quantum state is manipulated by applying resonant microwave pulses (phase χ) on the gate, or adiabatic pulses on the bias-current. These pulses induce a rotation of the effective spin $\overrightarrow{S}$ representing the qubit state on the Bloch sphere in the rotating frame. The Rabi precession of $\overrightarrow{S}$ during a microwave pulse results in oscillations of the switching probability p with the pulse length τ.

FIG. 2: Switching probability after two $\pi/2$ pulses with detuning $\Delta\nu = +52$ MHz, and with different phases corresponding to rotation axes $X, Y, -X$, or $-Y$, as a function of the delay between the pulses. The solid lines are fits including a finite decay time of 250 ns. The Ramsey patterns are phase-shifted as predicted for the different combinations of rotation axes.

FIG. 3: Demonstration of rotations around the Z axis. A triangular bias-current pulse applied between the two pulses of a Ramsey sequence induces a frequency change, and thus a phase shift between the two qubit states. This phase-shift, equivalent to a rotation around the Z axis, results in oscillations of the switching probability p (symbols) with the pulse amplitude ΔI. The fit uses the measured dependence of the transition frequency with the phase δ.

FIG. 4: Demonstration of the robustness of a composite pulse with respect to frequency detuning: Switching probability after a $CORPSE$ $\pi(X)$ sequence(disks), and after a single $\pi(X)$ pulse (circles). The dashed line is the prediction for the $CORPSE$ $\pi(X)$ sequence, the arrow indicates the qubit transition frequency. The $CORPSE$ sequence works over a larger frequency range. The Rabi frequency was 92 MHz. Inset: oscillations of the switching probability after a single pulse $\theta(-X)$ followed (disks) or not (circles) by a $CORPSE$ $\pi(X)$ pulse. The patterns are phase shifted by π, which shows tthat the $CORPSE$ sequence does correspond to a not operation.

FIG. 5: Top panel: switching probability (dots) after a Ramsey $\{\pi/2(X),\ \pi/2(X)\}$ sequence at $\Delta\nu = +50$ MHz, as a function of the time delay between pulses. The lines are exponential fits of the envelope with a time constant $T_2 = 350$ ns. This decay time actually varies due to changes in the charge fluctuators. Middle panel: example of echo measured in a $\{\pi/2(X),\ \pi(X),\ \pi/2(X)\}$ sequence (dots). The arrow indicates the nominal position of the echo minimum. Thin line: echo signal at the nominal minimum position. The bold line is an exponential fit of the envelope with a 550 ns time constant. The dashed line shows a fit of the lower envelope of the Ramsey pattern measured in the same conditions (220 ns time constant). Bottom panel: switching probability (thin lines) after two spin-locking sequences with a Rabi locking frequency of 24 MHz, at the optimal working point, as a function of the sequence duration. Thick lines: exponential fits of the envelopes, with time constant 650 ns. The dashed lines show a fit of the envelope of the Ramsey pattern measured in the same conditions (time constant : 320 ns). The residual oscillations in echo and spin-locking signals are due to pulse sequence imperfections.

$\pi/2(X), \pi/2 (X)$

$\pi/2(X), \pi(X), \pi/2 (X)$

$\pi/2(X), Lock(Y), \pi/2 (X)$

$\pi/2(X), Lock(Y), 3\pi/2 (X)$

switching probability (%)

duration Δt (ns)